Lecture 10 2025-09-25

Last time: how to convert infinite-dimensional traj. opt. problem to a discretized form

NOTE: discretizing the traj. opt. problem makes the # of decision variables finite

→ this does not mean decision variables can only attain a discrete set of possible values

$$x_k \in \mathbb{R}^{n_x} \qquad u_k \in \mathbb{R}^{n_u}$$

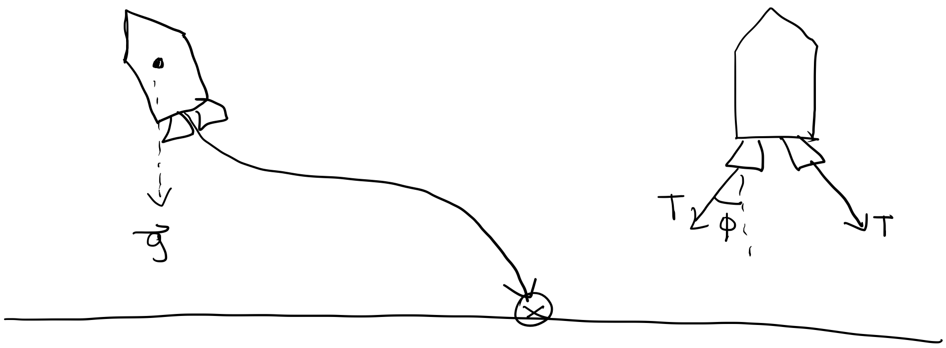
Today: apply this to powered descent guidance

We will follow the derivation provided by two papers:

1. B. Açikmese and S. Ploen, "Convex Programming Approach to Powered Descent Guidance for Mars Landing," in AIAA Journal of Guidance, Control, and Dynamics, vol. 30, no. 5, pp. 1353–1366, 2007.

Dynamics, vol. 30, no. 5, pp.

2. B. Acikmese, J. M. Carson, III, and L. Blackmore, "Lossless Convexification of Nonconvex Control Bound and Pointing Constraints of the Soft Landing Optimal Control Problem," in IEEE Transactions on Control Systems Technology, vol. 21, no. 6, pp. 2104–2113, 2013.



ϕ: cant angle (angle between T and $\hat{z}_B$)

I_{sp}: specific impulse (measure of efficiency of the rocket engine

g: gravity

n: # of thrusters

m_{wet}: "wet" mass of rocket when fueled

r_v : altitude

r_d : downrange position vector

γ: glideslope angle $\rightarrow \gamma \in (0, \pi/2) \rightarrow \beta = \tan \gamma$

Trajectory optimization problem:

state: position $r(t) \in \mathbb{R}^3$

velocity $\dot{r}(t) \in \mathbb{R}^3$

mass $\quad m(t) \in \mathbb{R}$

[P] $\quad \min_{t_F,\, T_c(t)} \int_0^{t_F} \|T_c(t)\|_2 \, dt$ "minimize thruster burn"

subj. to: $\ddot{r}(t) = g + \frac{\|T_c(t)\|_2}{m(t)}$

$$\dot{m}(t) = -a \|T_c(t)\|_2$$

$$0 \leq p_1 < T_c(t) \leq p_2$$ "thruster firing constraint"

$p_1 = n T_1 \cos\phi \qquad p_2 = n T_2 \cos\phi$

$$\|r_d(t)\|_2 \leq \beta r_v(t)$$ glideslope constraint

$m(0) = m_{wet} \quad m(t_F) \geq 0$

$r(0) = r_{init} \quad r(t_F) = 0$

$\dot{r}(0) = \dot{r}_{init} \quad \dot{r}(t_F) = 0$

boundary constraints

→ this is a non-convex, free final time, fixed final state problem

Convexify in two steps:

1. Deal with non-convex thruster firing constraint by "lifting" problem

2. Change-of-variables for mass flow

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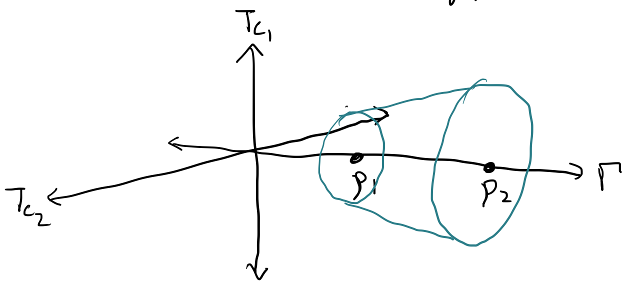
1. $0 < p_1 \leq \|T_c(t)\|_2 \leq p_2$

In 2D:

Lift to 3D and impose: $\|T_c(t)\|_2 \le \Gamma(t)$

$$0 < \rho_1 \le \Gamma(t) \le \rho_2$$



→ rewrite $\mathcal{P}$:

[$\bar{\mathcal{P}}$]

$$\min_{t_F,\, T_c(\cdot),\, \Gamma(\cdot)} \int_0^{t_F} \Gamma(t)\, dt$$

subj. to: $\ddot{r}(t) = g + \frac{T_c(t)}{m(t)}$ ← still non-convex w.r.t. $m(t)$

$$\dot{m}(t) = -\alpha \Gamma(t)$$

$$0 < \rho_1 \le \Gamma(t) \le \rho_2$$

$$\|T_c(t)\|_2 < \Gamma(t)$$

$$\|r_d(t)\|_2 \le \beta r_v(t)$$

(same boundary conditions as $\mathcal{P}$)

2. Use change of variables to convexify $m(t)$

$$\sigma(t) = \frac{\Gamma(t)}{m(t)} \qquad u(t) = \frac{T_c(t)}{m(t)}$$

$\rightarrow \Gamma(t) = m(t)\sigma(t)$

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$\rightarrow$ can rewrite dynamics as

$$\ddot{r}(t) = g(t) + u(t)$$

now reformulate to omit $m(t)$ from decision variables

$$\dot{m} = -\alpha \Gamma(t) = -\alpha\, m(t)\, \sigma(t)$$

$$\rightarrow \frac{\dot{m}(t)}{m(t)} = -\alpha\, \sigma(t)$$

introduce $z(t) = \ln m(t)$ (since $m(t) > 0$)

$$\dot{z}(t) = \frac{\dot{m}(t)}{m(t)} = -\alpha\, \sigma(t)$$

$$\rightarrow \dot{z}(t) = -\alpha\, \sigma(t)$$

also had: $0 < p_1 \leq \Gamma(t) \leq p_2$

$$\sigma(t) = \frac{\Gamma(t)}{m(t)} \rightarrow \Gamma(t) = \sigma(t)\, m(t)$$

$$\rightarrow 0 < p_1 \leq \sigma(t)\, m(t) \leq p_2$$

$$\rightarrow 0 < \frac{p_1}{m(t)} \leq \sigma(t) \leq \frac{p_2}{m(t)}$$

since $z(t) = \ln m(t)$

$$m(t) = e^{z(t)} \Leftrightarrow \frac{1}{m(t)} = e^{-z(t)}$$

$$\rightarrow 0 < \underbrace{p_1 e^{-z(t)} \leq \sigma(t)} \leq p_2 e^{-z(t)}$$

$p_1 e^{-z(t)} \le \sigma(t)$ is convex

$\sigma(t) \le p_2 e^{-z(t)}$ is non-convex

→ use Taylor series approximations of this constraint

let $\mu_1 = p_1 e^{-z_0}$ and $\mu_2 = p_2 e^{-z_0}$

where $z_0(t) = \ln(m_{wet} - \alpha p_2 t)$

$$\to \mu_1(t)\left[1 - (z(t) - z_0(t)) + \tfrac{1}{2}(z(t) - z_0(t))^2\right] \le \sigma(t)$$

$$\sigma(t) \le \mu_2(t)\left[1 - (z(t) - z_0(t))\right]$$

→ now we have convex approximation:

$$[\tilde{P}] \qquad \min_{T_f, \sigma(t), u(t)} \int_{T_0}^{T_f} \sigma(t)\, dt$$

subj. to: $\ddot{r}(t) = u(t) + g$

$\dot{z}(t) = -\alpha \sigma(t)$

$\|u(t)\|_2 \le \sigma(t)$

$$\mu_1(t)\left[1 - (z(t) - z_0(t)) + \frac{(z(t) - z_0(t))^2}{2}\right] \le \sigma(t) \le \mu_2(t)\left[1 - (z(t) - z_0(t))\right]$$

$$z_0(t) \le z(t) \le \ln(m_{wet} - \alpha p_1 t)$$

$$\|r_d(t)\|_2 \le \beta r_v(t)$$

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$$z_0(t) = \ln(m_{wet} - \alpha p_2 t)$$

$$\mu_1(t) = p_1 e^{-z_0(t)} \qquad \mu_2(t) = p_2 e^{-z_0(t)}$$

$$z_0(0) = m_{wet}$$

$$r(0) = r_{mrt} \qquad r(t_f) = 0$$

$$\dot{r}(0) = \dot{r}_{mrt} \qquad \dot{r}(t_f) = 0$$

3. Now discretize this problem and call convex solver

Assumptions

1. Deterministic model
2. Omits rotational kinematics/dynamics
3. Perfect closed-loop tracking controller
4. Missing drag physics

"All models are wrong, but some are useful"